Beyond Worst-case Analysis of Multicore Caching Strategies

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Introduction

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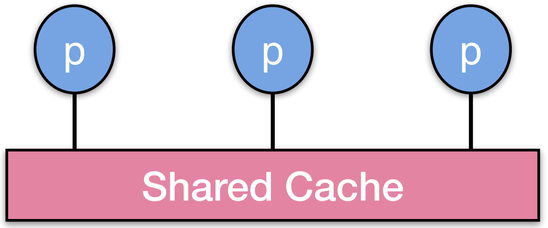

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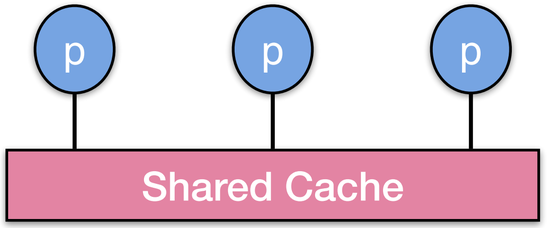

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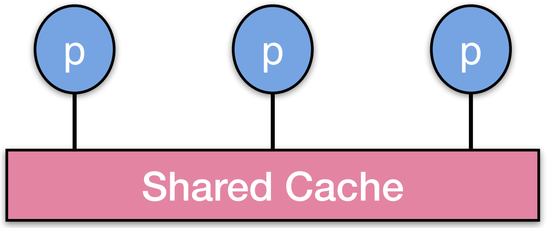

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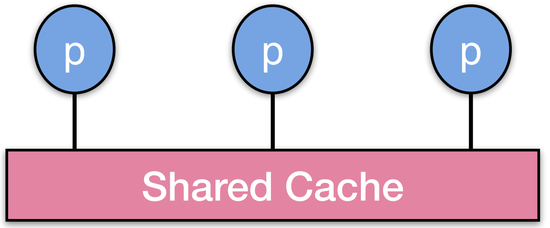

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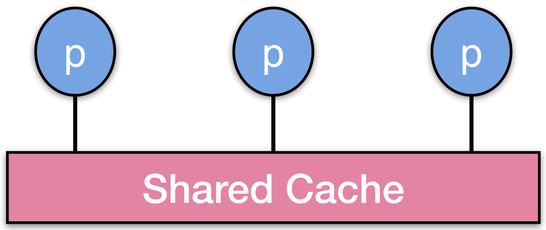
A large class of “lazy algorithms” (including LRU) **cannot be competitive** in multicore caching [KX20].

Relaxing Competitive Analysis

Resource augmentation [ST85] relaxes strict worst-case (competitive) analysis by giving online algorithms additional resources.



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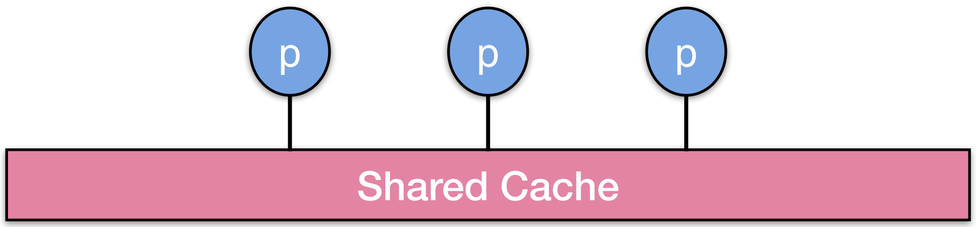

Online algorithm

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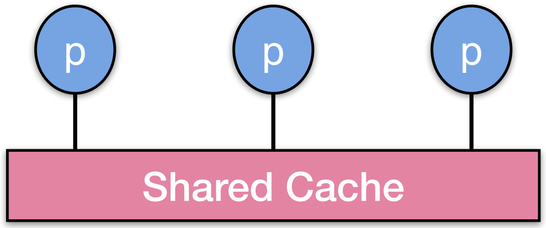

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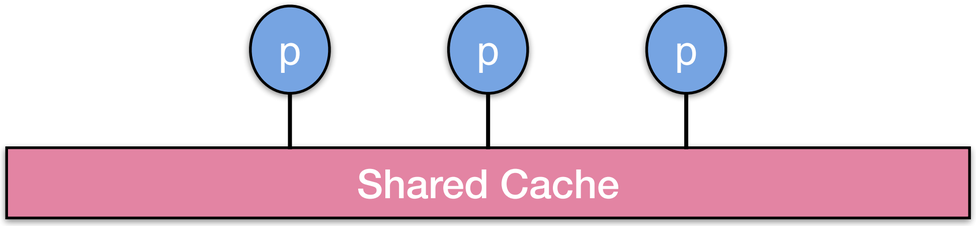

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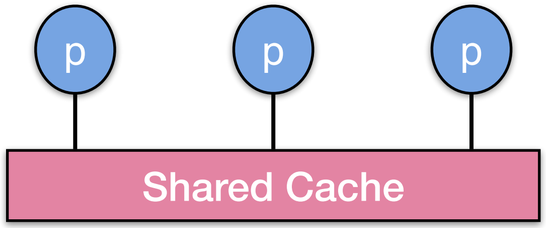
In multicore caching, a constant factor in resource augmentation yields an **O(log p)-competitive** algorithm (where p is the number of cores) [ABDKPS20].



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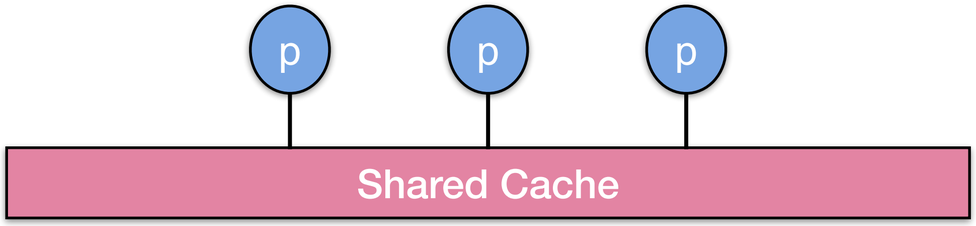

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Alternative Measures

Many alternative measures have been proposed for single-core caching but have not been studied in the **parallel setting** [Y94, Y98, KP00, KPR92, BLN01, BFL05, BB94].

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Overly pessimistic

Bijective analysis 💚

Competitive analysis 😡

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Cyclic Analysis for Multicore Caching

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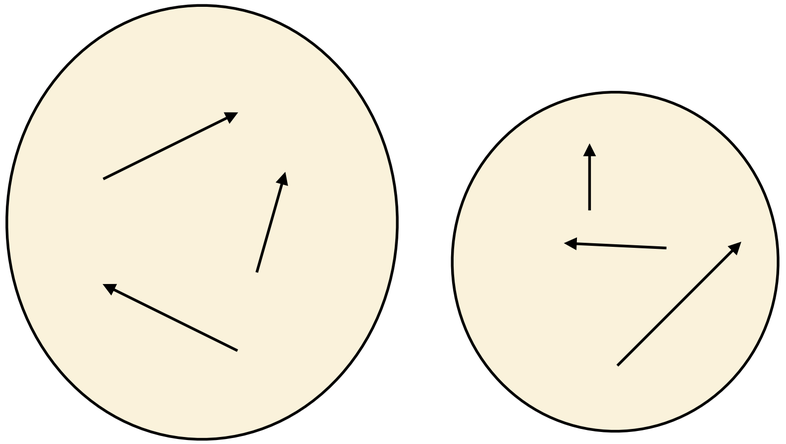
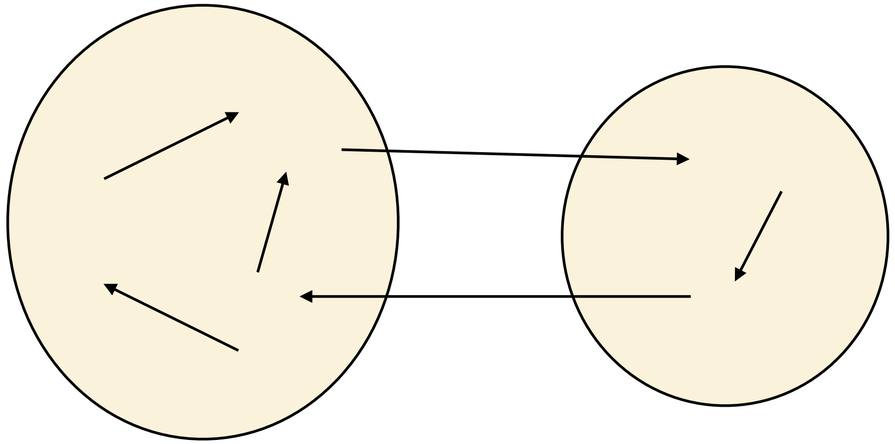
Bijective analysis compares inputs of the same “length”, while cyclic analysis compares algorithms **across all inputs** rather than within partitions.



Bijective analysis



Cyclic analysis

Cyclic Analysis Measure

	Applicability
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Cyclic Analysis Measure

	Applicability	Compares online algorithms directly
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Cyclic Analysis Measure

Used to separate LRU

	Applicability	Compares online algorithms directly	Can incorporate restrictions on universe of inputs
Bijective analysis	😡	💚	💚
Cyclic analysis	💚	💚	💚

Separating LRU With Cyclic Analysis

An online algorithm A is **no worse** than online algorithm B under **cyclic analysis** on the universe of inputs I if there exists a bijection $\pi : I \leftrightarrow I$ satisfying $A(R) \leq B(\pi(R))$ for all $R \in I$. We denote this by $A \leq_c B$.

Algorithm cost

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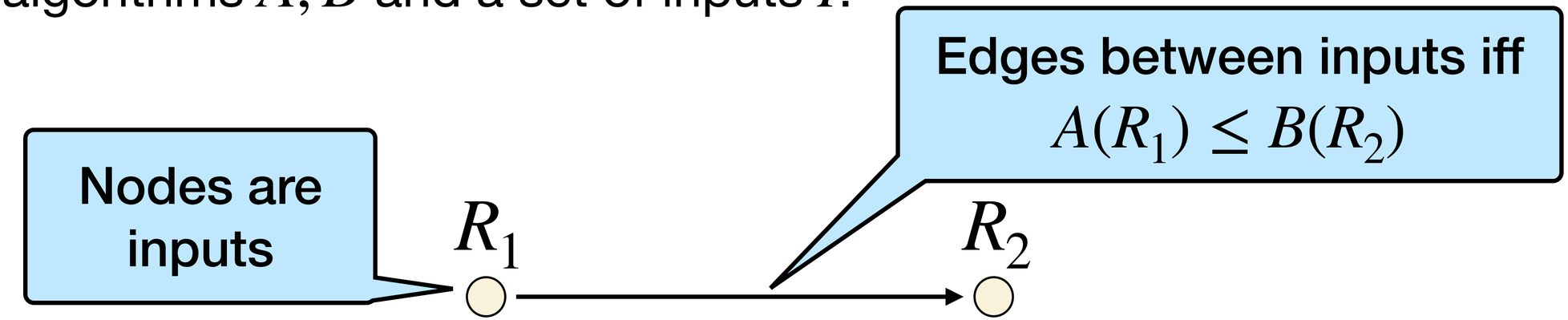
Main theorem: For any lazy caching algorithm A,

$LRU \leq_c A$ **on inputs with locality of reference.**

Cyclic Analysis: A Tutorial

Input-cost graph: modeling mappings between inputs with different costs.

Given algorithms A, B and a set of inputs I:



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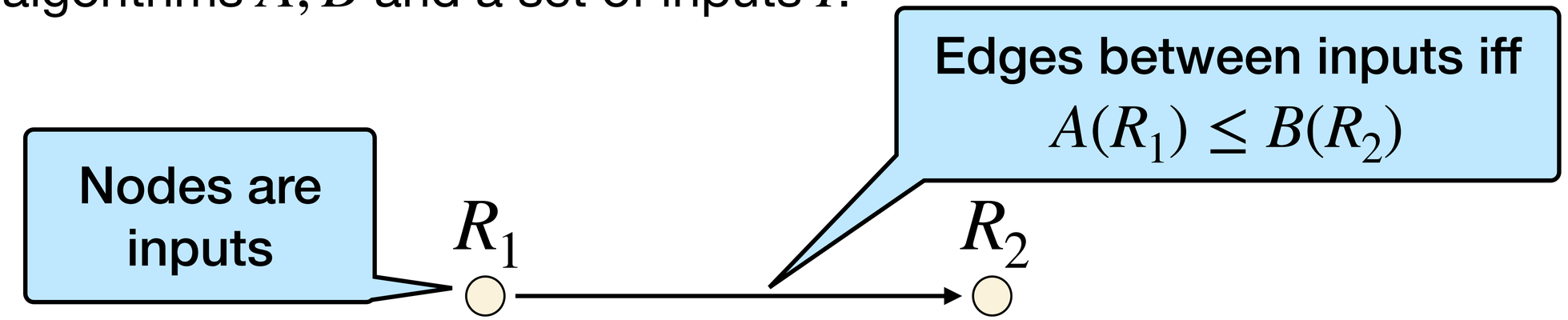

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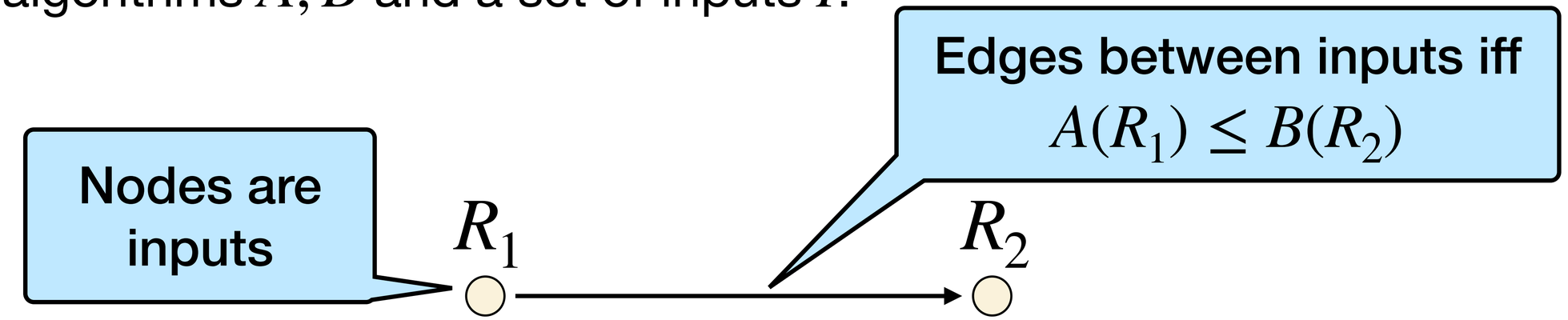

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Analysis technique: Partition the input-cost graph and analyze cycles in the resulting subgraphs.

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Partitioning the input-cost graph: For problems such as single-core caching, the length of inputs is well defined.

For general online problems (e.g. multicore caching), the length is not a measure of difficulty.

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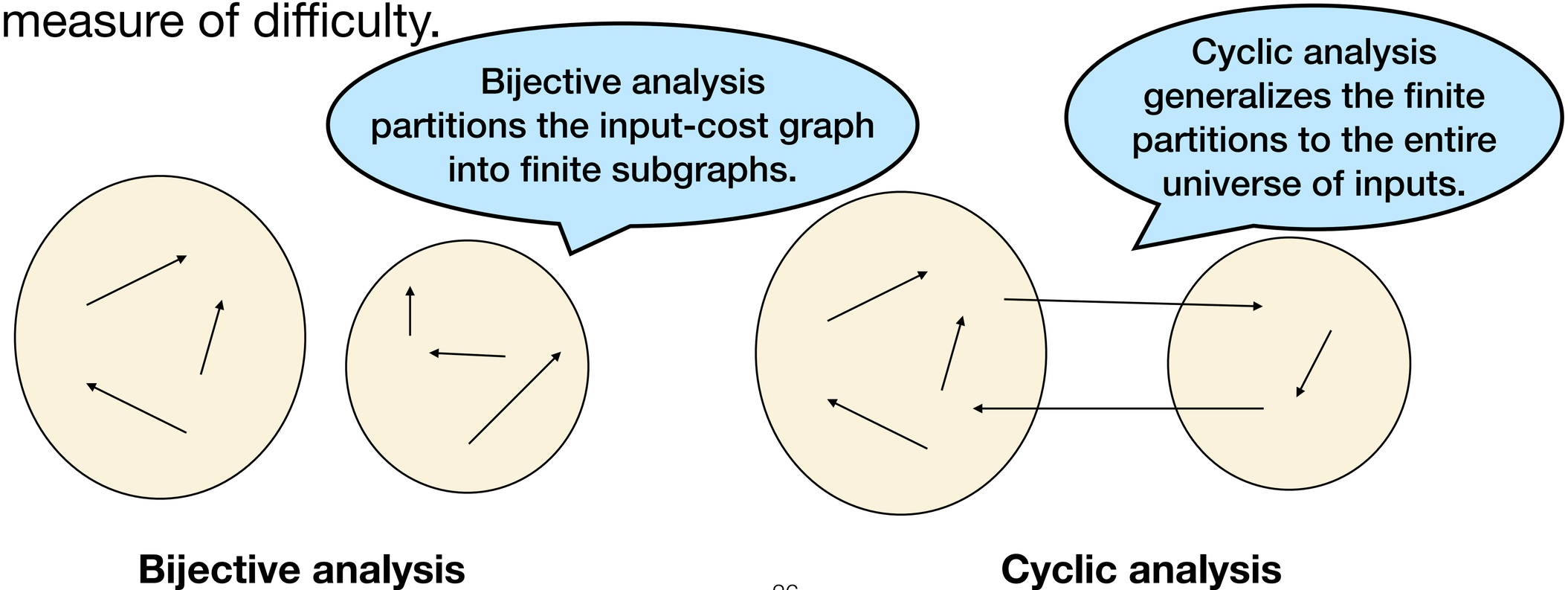

Example: Bijective vs Cyclic Analysis

Given algorithms A, B for some online problem P.

$A(R_1) = 10$	$B(R_1) = 20$
$A(R_2) = 40$	$B(R_2) = 30$
$A(R_3) = 20$	$B(R_3) = 40$
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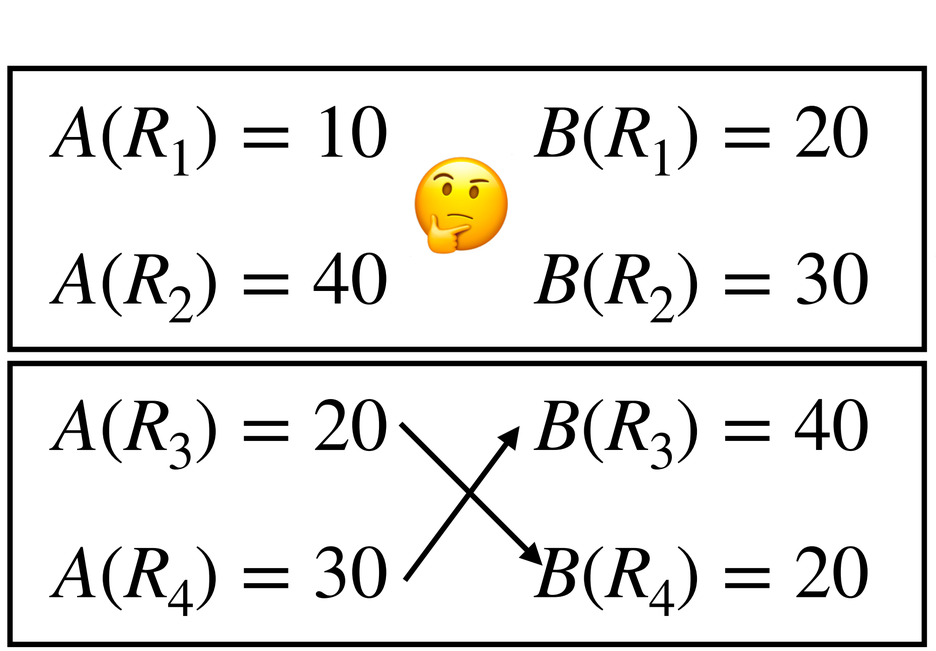

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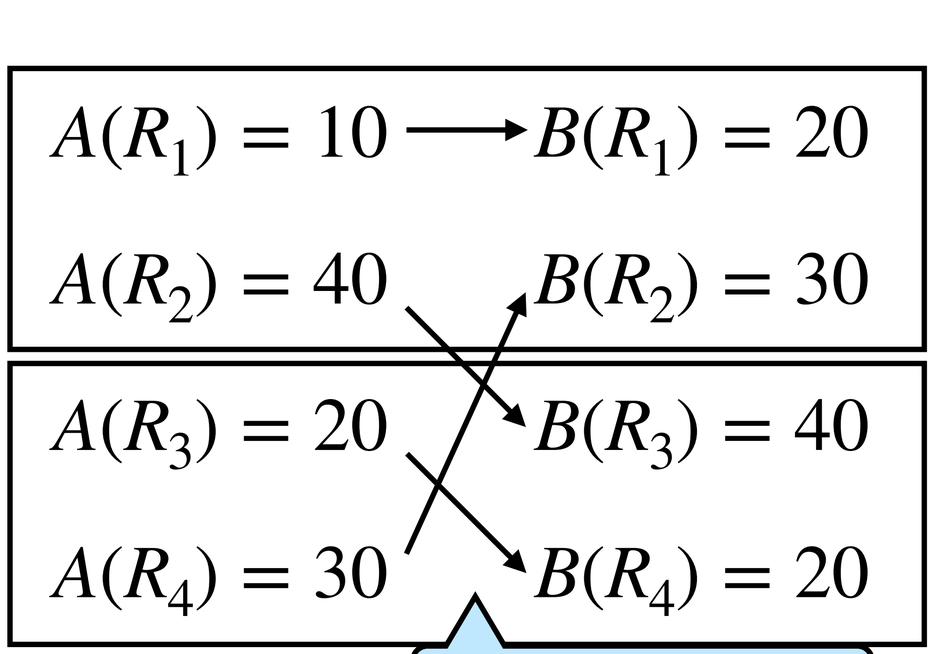

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Under cyclic analysis:

$A <_C B$

Properties of Cyclic Analysis

The cyclic analysis measure has several **natural properties**:

Transitivity ($A \leq_c B, B \leq_c C \rightarrow A \leq_c C$).

Directedness ($A <_c B$ implies $B \not\leq_c A$).

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Cyclic analysis can be used to show **“to-be-expected” relationships** between multicore caching algorithms:

All **lazy algorithms are equivalent**.

Any lazy algorithm A is **strictly better than Flush-When-Full** (for p=2).

Free-Interleaving Model [LS12]

Problem setup: given a multicore processor with p cores and k pages.

An input R is formed by p request sequences $R = R_1, \ldots, R_p$ composed of page requests, where each core must serve its corresponding request sequence.

Page requests arrive at discrete timesteps.

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Cache: ab

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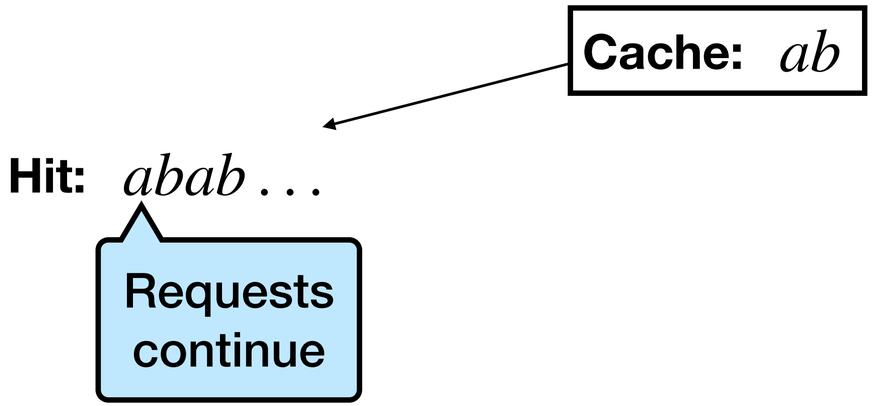

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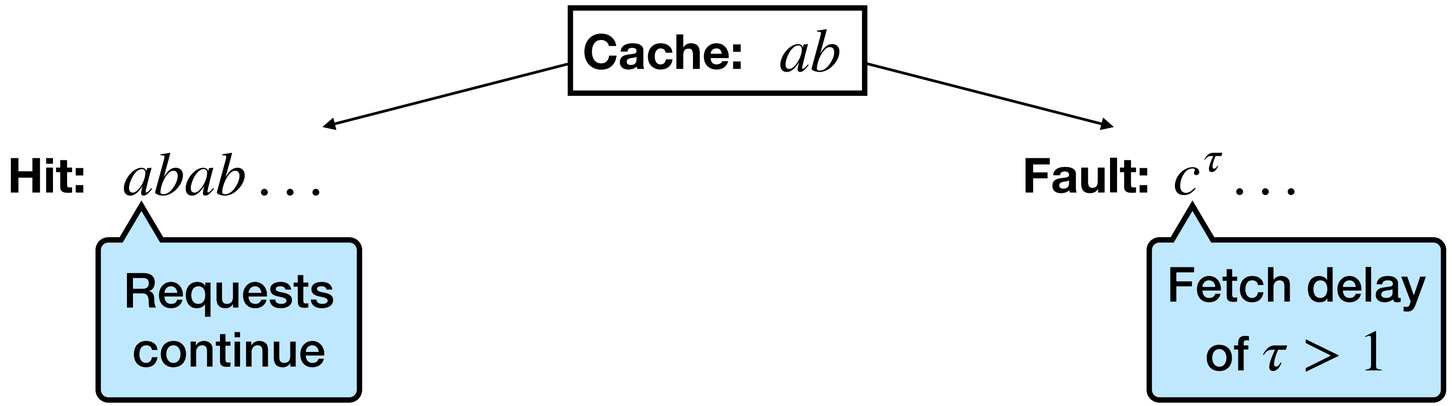

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$abcde\ldots$

No locality

😡

$ababa\ldots$

Some locality

💚

Advantage of LRU with Locality

Goal: For any lazy algorithm *A*, define a bijection π over the space of inputs with locality I satisfying $LRU(R) \leq A(\pi(R))$ for all $R \in I$.

Intuition: Analyze timesteps where A and LRU differ. Suppose at time t, LRU evicts page σ_{LRU}, while A evicts σ_{NLRU}.

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$\pi(R)$ swaps them in the continuation.
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If there was not a miss before a request to σ_{NLRU} after time *t*, LRU hits on the first request to σ_{NLRU}.

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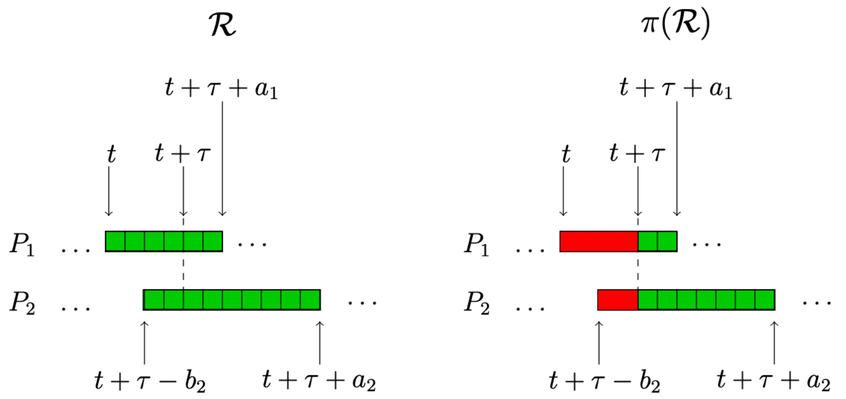
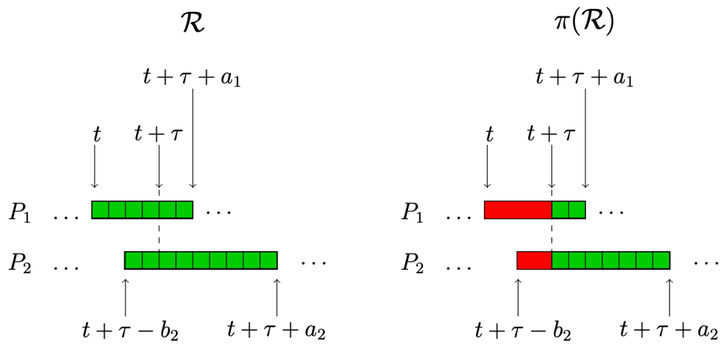
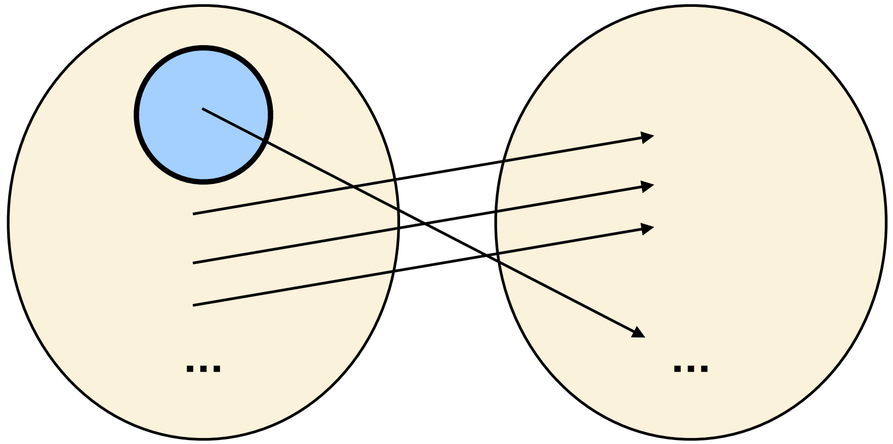


Mapping to Inputs of Different Length





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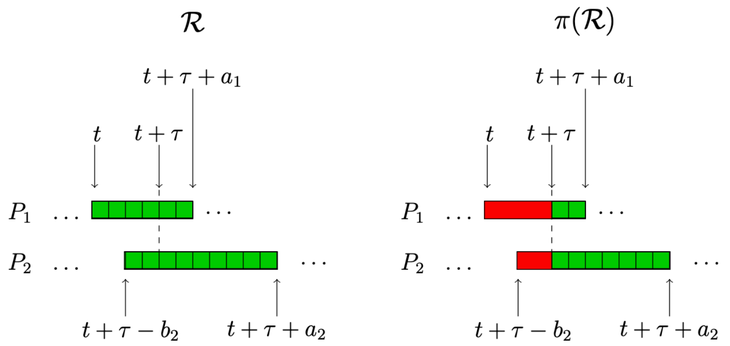

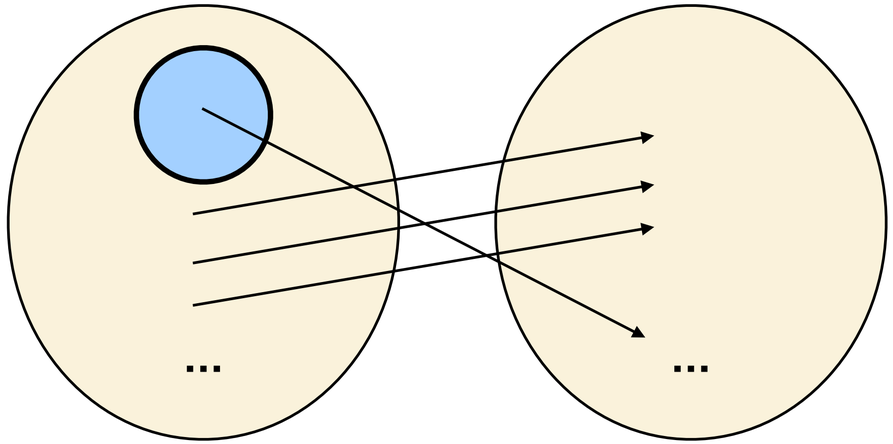

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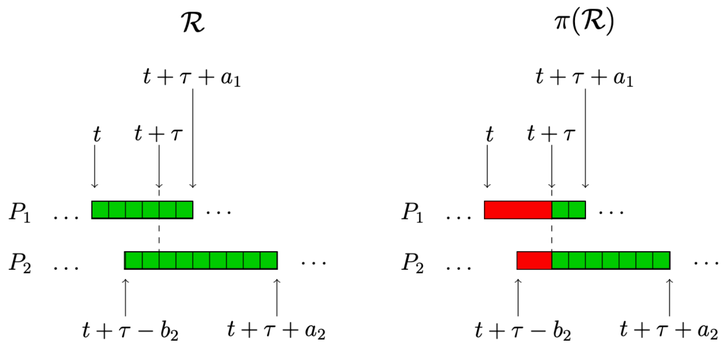

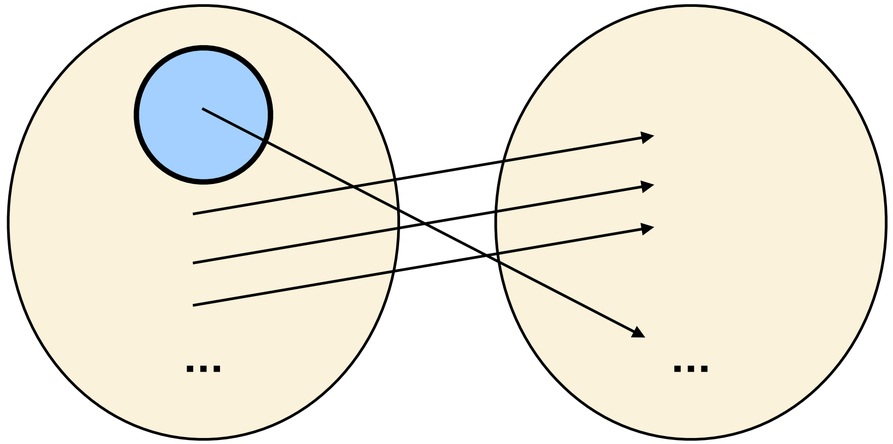

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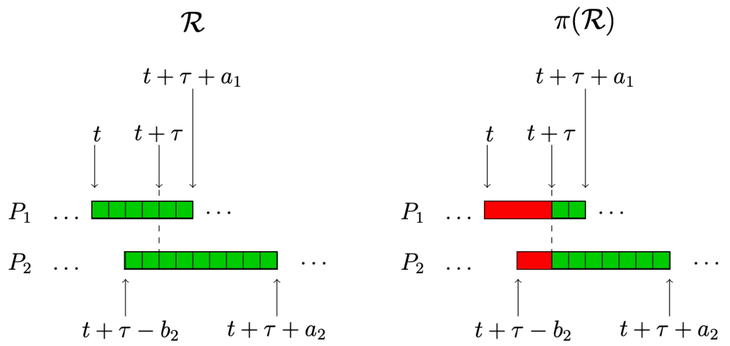

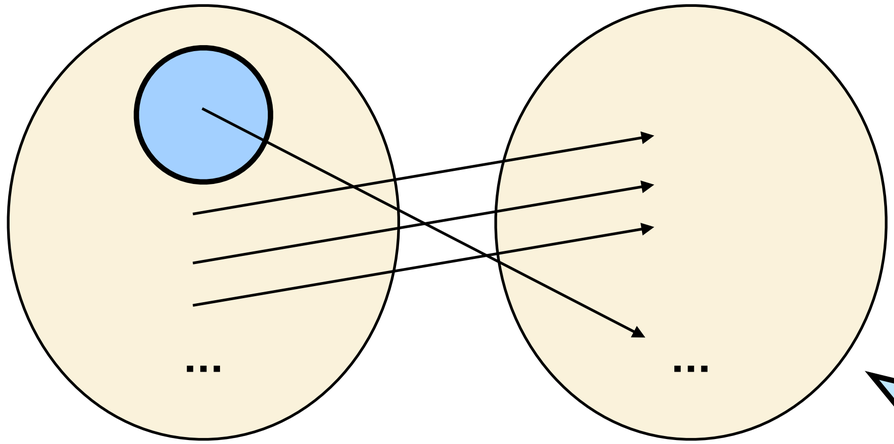

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Multicore caching is **well-studied in practice** and motivated by hierarchical memory.

The need for **theoretical understanding** will only grow as multicore architectures become more prevalent.